Five Year Integrated M.Sc. Examination, 2022-2023
Semester - V
Course: PH-3-5-1
Time: Four Hours (Classical Mechanics) Full Marks: 80

Questions are of values as indicated in the margin
Answer question number **one** and any **four** from the rest

1. Answer any **five** questions:

$5 \times 4 = 20$

(a) Consider a mass m, moving about an axis on a plane. This system is acted upon by a central force $\vec{F}(\vec{r})$.

i. Show that the angular momentum, $\vec{L}$ of the mass is a constant of motion.

ii. What is the definition of a cyclic coordinate of a dynamical system?

iii. Consider the present system. Does the appropriate Lagrangian $\mathcal{L}$ have any cyclic coordinate?

2+1+1=4

(b) Consider a conservative dynamical system with n degrees of freedom. So, if $\vec{q}$ and $\vec{p}$ are generalized coordinates and generalized momenta respectively, $\vec{q} = (q_1, q_2, \ldots, q_n)$ and $\vec{p} = (p_1, p_2, \ldots, p_n)$. Let this system has the Lagrangian $\mathcal{L}(\dot{\vec{q}}, \vec{q})$ and the Hamiltonian $\mathcal{H}(\vec{q}, \vec{p})$.

i. Find the relation between $\vec{p}$ and $\mathcal{L}(\dot{\vec{q}}, \vec{q})$.

ii. What is the expression of the Hamiltonian, $\mathcal{H}(\vec{q}, \vec{p})$ in terms of $\vec{q}$, $\dot{\vec{q}}$, $\vec{p}$ and $\mathcal{L}(\dot{\vec{q}}, \vec{q})$?

iii. Using the equations of motion in the Lagrangian formalism, find the equations of motion in the Hamiltonian formalism.

1+1+2=4

(c) Consider Kepler's problem i.e. the problem of the motion of a particle of unit mass is attracted by an inverse square gravitational force towards a fixed points.

i. Write down its Hamiltonian in polar coordinates.

ii. Find the equations of motion. No explicit solution is required.

2+2=4

(d) i. One of the rails of a railroad, except at the equator wears out on the inside noticeably more than the other - Explain.

ii. It is found that in the Northern Hemisphere the right bank of the river washed away more than its left bank - Explain.

iii. At the equator a body falls from a height of 80 meters. Find the Coriolis deflection. ($g \simeq 9.8\, m/s^2$).

1+1+2=4

(e) i. Suppose a bicycle rider negotiates a sharp bend. State what would be the prudent way for negotiation. Explain if through a vectorial representation.

ii. What is a wheel of laugh? If you ride a wheel of laugh, what would be the safe place for you? Will a paraboloid wheel of laugh be better than a plane of wheel of laugh. Explain the reasons.

2+2=4

(f) Consider the two-dimensional polar coordinate system, characterized by unit vectors $\vec{e_r}$ and $\vec{e_\theta}$.

i. Find the expression of these vectors, $\vec{e_r}$ and $\vec{e_\theta}$ in the Cartesian coordinate system.

ii. Find time derivative of these vectors, i.e. $\dot{\vec{e_r}}$ and $\dot{\vec{e_\theta}}$.

iii. A unit small moves on a circle of radius r with a constant speed v. Find the acceleration of this mass. Also point out the direction of acceleration.

1+1+2=4

2. (a) A bead slides without friction on a rigid wire rotating at a constant angular speed ω. Find the force exerted by the wire on the bead.

(b) A particle of mass m is constrained to move on the inside surface of a smooth cone of half angle, α. Determine a set of generalized coordinates and the constraint, if any. Find Lagrange's equations of motion.

10+10=20

3. (a) Define holonomic and nonholonomic constraints with examples.
   (b) State and explain Hamilton's least action principle.
   (c) Derive Lagrange's equation from the Hamilton's least action principle.
   (d) Two masses, m_1 and m_2 are tied to the ends of an inextensible string of length l passing over a fixed pulley at the edge of a smooth table. Set up the Lagrange's equations and Hamilton's equations.

4+2+7+7=20

4. Consider a one-dimensional pendulum, made up of a bob of mass m and an inextensible string of length, l which is hung from a pivot.
   (a) Write down the Lagrangian and the Hamiltonian of this dynamical system with a pictorial representation of the model.
   (b) Write down the equations of motion.
   (c) Find its fixed points, noting that it has two types of fixed points.
   (d) Draw the potential diagram of the model and on this diagram mark position of these fixed points.
   (e) You know that the model has two sets of fixed points, centres and saddle points. Mathematically analyse the dynamical behaviour of the model around at least one type of fixed points.

2+2+2+2+12=20

5. Consider the problem of spherical pendulum.
   (a) From Lagrange's equations of the first kind, find equations of motion in the Cartesian coordinate system.
   (b) Find constants of motion from these equations.
   (c) What would be the appropriate generalised coordinates for this system.
   (d) Derive dynamical equations in generalised coordinates from the Cartesian set of equations.

4+4+2+10=20